String matching

Data Structures and Algorithms for Computational Linguistics III
(ISCL-BA-07)

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Seminar für Sprachwissenschaft

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Finding patterns in a string

- Finding a pattern in a larger text is a common problem in many applications
- Typical example is searching in a text editor or word processor
- There are many more:
  - DNA sequencing / bioinformatics
  - Plagiarism detection
  - Search engines / information retrieval
  - Spell checking
  - ...

Types of problems

- The efficiency and usability of algorithms depend on some properties of the problem
- Typical applications are based on finding multiple occurrences of a single pattern in a text, where the pattern is much shorter than the pattern
- The efficiency of the algorithms may depend on the
  - relative size of the pattern
  - expected number of repetitions
  - size of the alphabet
  - whether the pattern is used once or many times
- Another related problem is searching for multiple patterns at once
- In some cases, fuzzy / approximate search may be required
- In some applications, preprocessing (indexing) the text to be searched may be beneficial

Problem definition

and some terminology

text:	A	A	T	A	G	A	C	G	G	C	T	A	G	C	A	A
pattern:				A	G	A	C									

- We want to find all occurrences of pattern p (length m) in text t (length n)
- The characters in both t and p are from an alphabet Σ, in the example $\Sigma = \{A, C, G, T\}$
- The size of the alphabet (q) is often an important factor
- p occurs in t with shift s if $p[0 : m] == t[s : s + m]$, we have a match at $s = 3$ in the example
- A string x is a prefix of string y, if $y = xw$ for a possibly empty string w
- A string x is a suffix of string y, if $y = wx$ for a possibly empty string w

Brute-force string search

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
t:	A	A	T	A	G	A	C	G	G	C	T	A	G	C	A	A
p:	A	G	C	A												
j	0	1	2	3												

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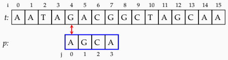

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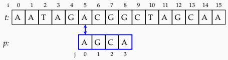

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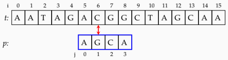

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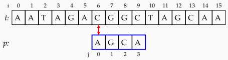

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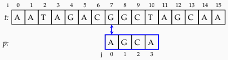

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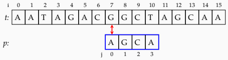

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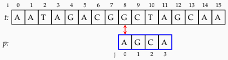

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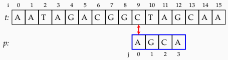

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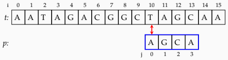

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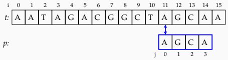

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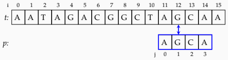

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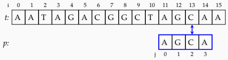

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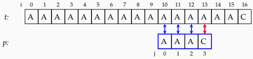

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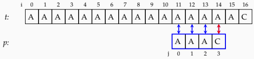

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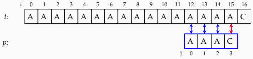

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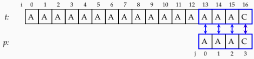
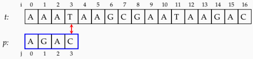


- Worst-case complexity of the method is $O(nm)$
- Crucially, most of the comparisons are redundant
  - for $i > 0$ and any comparison with $j = 0, 1, 2$, we already inspected corresponding i values
- The main idea for more advanced algorithms is to avoid this unnecessary comparisons

Boyer-Moore algorithm
slightly simplified version



- The main idea is to start comparing from the end of p
- If t[i] does not occur in p, shift m steps
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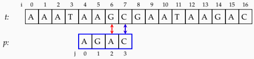

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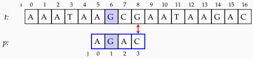

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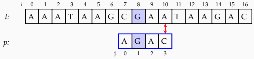

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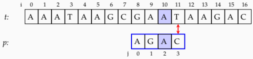

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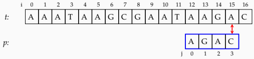

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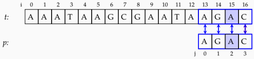
- Otherwise, align the last occurrence of t[i] in p with t[i]

Boyer-Moore algorithm
slightly simplified version



- The main idea is to start comparing from the end of p
- If t[i] does not occur in p, shift m steps
- Otherwise, align the last occurrence of t[i] in p with t[i]

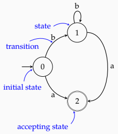
Boyer-Moore algorithm
implementation and analysis

- On average the algorithm performs better than brute-force
- In worst case the complexity of the algorithm is $O(nm)$, example: $t = aaa \ldots a, p = baa \ldots a$
- Faster versions exist $(O(n + m + q))$

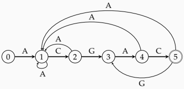
```
last = {}
for j in range(m):
    last[P[j]] = j
i,j = m-1, m-1
while i < n:
  if T[i] == P[j]:
    if j == 0:
      return i
    else:
      i -= 1
      j -= 1
  else:
    k = last.get(T[i], -1)
    i += m + min(j, k+1)
    j = m - 1
return None
```

A quick introduction to FSA

- Another efficient way to search a string is building a finite state automaton for the pattern
- An FSA is a directed graph where edges have labels
- One of the states is the *initial state*
- Some states are accepting states
- We will study FSA more in-depth soon



An FSA for the pattern ACGAC

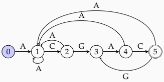


- Start at state 0, switch states based on the input
- All unspecified transitions go to state 0
- When at the accepting state, announce success

FSA pattern matching

demonstration

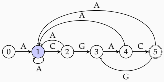
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	A	A	C	G	A	C	G	A	C	A	T	A	C	G	A	C



FSA pattern matching

demonstration

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	A	A	C	G	A	C	G	A	C	A	T	A	C	G	A	C



FSA pattern matching

demonstration

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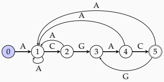
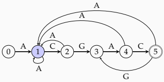


FSA pattern matching

demonstration

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	A	A	C	G	A	C	G	A	C	A	T	A	C	G	A	C



FSA pattern matching

demonstration

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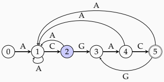

FSA pattern matching

demonstration

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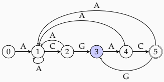

FSA pattern matching

demonstration

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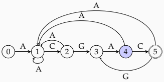

FSA pattern matching

demonstration

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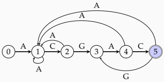

FSA pattern matching

demonstration

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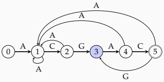

FSA pattern matching

demonstration

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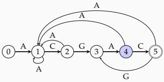
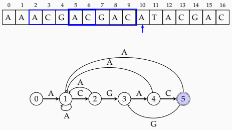


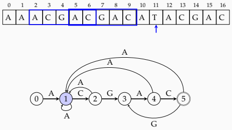
FSA pattern matching

demonstration



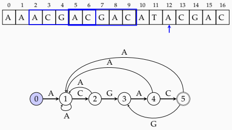
FSA pattern matching

demonstration



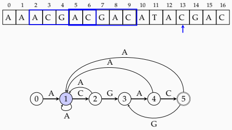
FSA pattern matching

demonstration



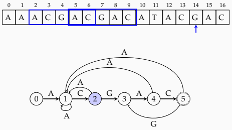
FSA pattern matching

demonstration



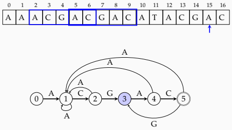
FSA pattern matching

demonstration



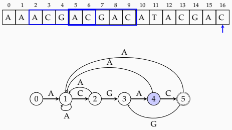
FSA pattern matching

demonstration



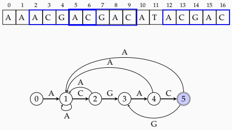
FSA pattern matching

demonstration



FSA pattern matching

demonstration



FSA for string matching

how to build the automaton

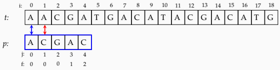
- An FSA results in $O(n)$ time matching, however, we need to first build the automaton
- At any state of the automaton, we want to know which state to go for the failing matches
- Given substring s recognized by a state and a non-matching input symbol a, we want to find the longest prefix of s such that it is also a suffix of sa
- A naive attempt results in $O(qm^3)$ time for building the automaton (where q is the size of the alphabet m is the length of the pattern)
- If stored in a matrix, the space requirement is $O(m^2)$
- Better (faster) algorithms exist for construction these automaton (we will cover some later in this course)

Knuth-Morris-Pratt (KMP) algorithm

- The KMP algorithm is probably the most popular algorithm for string matching
- The idea is similar to the FSA approach: on failure, continue comparing from the longest matched prefix so far
- However, we rely on a simpler data structure (a function/table that tells us where to back up)
- Construction of the table is also faster

KMP algorithm

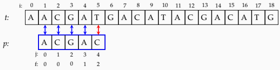
demonstration



- In case of a match, increment both i and j
- On failure, or at the end of the pattern, decide which new $p[j]$ compare with $t[i]$ based on a function f
- $f[j-1]$ tells which j value to resume the comparisons from

KMP algorithm

demonstration



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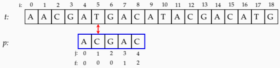
KMP algorithm

demonstration



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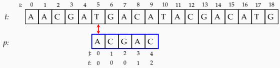
KMP algorithm

demonstration



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KMP algorithm

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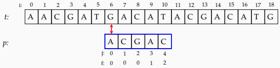

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KMP algorithm

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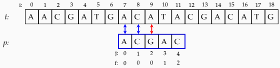

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KMP algorithm

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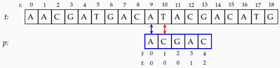

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KMP algorithm

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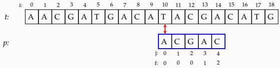

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KMP algorithm

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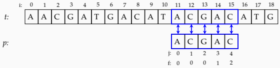

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KMP algorithm

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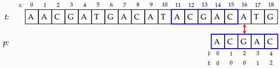

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- f[j − 1] tells which j value to resume the comparisons from

Complexity of the KMP algorithm

- In the while loop, we either increase i, or shift the comparison
- As a result, the loop runs at most 2n times, complexity is $O(n)$

```
i, j = 0, 0
while i < n:
  if T[i] == P[j]:
    if j == m - 1:
      return j - m + 1
    else:
      i += 1
      j += 1
  elif j > 0:
    j = fail[k - 1]
  else:
    j += 1
return None
```


Building the failure table

```
f = [0] * m
j, k = 1, 0
while j < m:
  if P[j] == P[k]:
    f[j] = k + 1
    j += 1
    k += 1
  elif k > 0:
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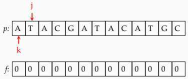

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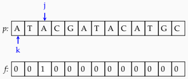

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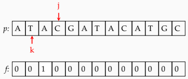

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  else:
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```

					j								
p:	A	T	A	C	G	A	T	A	C	A	T	G	C
	k												
f:	0	0	1	0	0	0	0	0	0	0	0	0	0

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```

						j							
p:	A	T	A	C	G	A	T	A	C	A	T	G	C
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```

							j						
p:	A	T	A	C	G	A	T	A	C	A	T	G	C
		k											
f:	0	0	1	0	0	1	2	0	0	0	0	0	0

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								j					
p:	A	T	A	C	G	A	T	A	C	A	T	G	C
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									j				
p:	A	T	A	C	G	A	T	A	C	A	T	G	C
				k									
f:	0	0	1	0	0	1	2	3	4	0	0	0	0

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```

										j			
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											j		
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		k											
f:	0	0	1	0	0	1	2	3	4	1	2	0	0

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												j	
p:	A	T	A	C	G	A	T	A	C	A	T	G	C
			k										
f:	0	0	1	0	0	1	2	3	4	1	2	0	0

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												j	
p:	A	T	A	C	G	A	T	A	C	A	T	G	C
		k											
f:	0	0	1	0	0	1	2	3	4	1	2	0	0

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    j += 1
    k += 1
  elif k > 0:
    k = fail[k - 1]
  else:
    j += 1
```

												j	
p:	A	T	A	C	G	A	T	A	C	A	T	G	C
	k												
f:	0	0	1	0	0	1	2	3	4	1	2	0	0

Building the failure table

```
f = [0] * m
j, k = 1, 0
while j < m:
  if P[j] == P[k]:
    f[j] = k + 1
    j += 1
    k += 1
  elif k > 0:
    k = fail[k - 1]
  else:
    j += 1
```

													j
p:	A	T	A	C	G	A	T	A	C	A	T	G	C
	k												
f:	0	0	1	0	0	1	2	3	4	1	2	0	0

Rabin-Karp algorithm

- Rabin-Karp string matching algorithm is another interesting algorithm
- The idea is instead of matching the string itself, matching the hash of it (based on a hash function)
- If a match found, we need to verify – the match may be because of a hash collision
- Otherwise, the algorithm makes a single comparison for each position in the text
- However, a hash should be computed for each position (with size m)
- Rolling hash functions avoid this complication

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9

$h = 39$

p: 4 3 8 5 7 9 4 3 $h(p) = 43$

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9

$h = 37$

p: 4 3 8 5 7 9 4 3 $h(p) = 43$

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

?

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9

$h = 43$

p: 4 3 8 5 7 9 4 3 $h(p) = 43$

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9

$h = 49$

p: 4 3 8 5 7 9 4 3 $h(p) = 43$

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9

$h = 47$

p: 4 3 8 5 7 9 4 3 $h(p) = 43$

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

?

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9

$h = 43$

p: 4 3 8 5 7 9 4 3 $h(p) = 43$

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Rabin-Karp string matching

demonstration with additive hashing

t: 7 1 3 6 7 4 3 8 5 7 9 4 3 9

$h = 48$

p: 4 3 8 5 7 9 4 3 $h(p) = 43$

- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

Summary

- String matching is an important problem with wide range of applications
- The choice of algorithm largely depends on the problem
- We will revisit the problem on regular expressions and finite-state automata
- Reading: Goodrich, Tamassia, and Goldwasser (2013, chapter 13)

Next:

- Algorithms on strings: edit distance / alignment, tries
- Reading: Goodrich, Tamassia, and Goldwasser (2013, chapter 13),Jurafsky and Martin (2009, section 3.11, or 2.5 in online draft)

Acknowledgments, credits, references

Goodrich, Michael T., Roberto Tamassia, and Michael H. Goldwasser (2013). *Data Structures and Algorithms in Python*. John Wiley & Sons, Incorporated. isbn: 9781118476734.

Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second edition. Pearson Prentice Hall. isbn: 978-0-13-504196-3.

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